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Enhanced sensitivity via non-Hermitian topology

Midya Parto^{1,2,3}, Christian Leefmans⁴, James Williams¹, Robert M. Gray¹ and Alireza Marandi^{1,4}✉

Abstract

Sensors are indispensable tools of modern life that are ubiquitously used in diverse settings ranging from smartphones and autonomous vehicles to the healthcare industry and space technology. By interfacing multiple sensors that collectively interact with the signal to be measured, one can go beyond the signal-to-noise ratios (SNR) attainable by the individual constituting elements. Such techniques have also been implemented in the quantum regime, where a linear increase in the SNR has been achieved via using entangled states. Along similar lines, coupled non-Hermitian systems have provided yet additional degrees of freedom to obtain better sensors via higher-order exceptional points. Quite recently, a new class of non-Hermitian systems, known as non-Hermitian topological sensors (NTOS) has been theoretically proposed. Remarkably, the synergistic interplay between non-Hermiticity and topology is expected to bestow such sensors with an enhanced sensitivity that grows exponentially with the size of the sensor network. Here, we experimentally demonstrate NTOS using a network of photonic time-multiplexed resonators in the synthetic dimension represented by optical pulses. By judiciously programming the delay lines in such a network, we realize the archetypal Hatano-Nelson model for our non-Hermitian topological sensing scheme. Our experimentally measured sensitivities for different lattice sizes confirm the characteristic exponential enhancement of NTOS. We show that this peculiar response arises due to the combined synergy between non-Hermiticity and topology, something that is absent in Hermitian topological lattices. Our demonstration of NTOS paves the way for realizing sensors with unprecedented sensitivities.

Introduction

The ability to accurately and reliably measure physical quantities is at the heart of modern sensors with applications ranging from molecular sensing in chemistry¹ and biology² to light detection and ranging (LiDAR)³ and observing gravitational waves⁴. Significant efforts have been made towards enhancing the responses of individual sensing elements, for instance, by using high-quality resonators⁵ or exploiting quantum effects⁶. A different, more generic route to achieving higher sensitivities is to employ a multitude of modes that collectively contribute to a coherent signal that encapsulates information about the quantity to be measured. This has led to classical and quantum sensing networks which allow for an enhancement of $\sqrt{N}$ and N^7 in the sensitivity figure, with N

denoting the number of independent/entangled entities contributing to the sensing signal, respectively.

An alternative path to achieve higher sensitivities is to employ concepts from non-Hermitian physics^{8–10}. The eigenvalues associated with a non-Hermitian system can respond to perturbations in a remarkably stronger manner compared to its Hermitian counterparts. This realization is the foundation of a class of sensors that operate in the vicinity of non-Hermitian degeneracies known as exceptional points (EPs), where in the presence of an N -th-order EP, the response scales as the N -th root of the perturbation^{11–16}. In addition, the introduction of non-Hermiticity to topologically non-trivial lattices is known to result in an eigenspace that behaves very differently from that associated with Hermitian topological systems^{17,18}. Recent studies have observed the manifestation of this distinct behavior in the form of a new type of bulk-boundary correspondence and the non-Hermitian skin effect^{19–25}.

Quite recently, a new class of sensors based on the synergy between non-Hermiticity and topology has been

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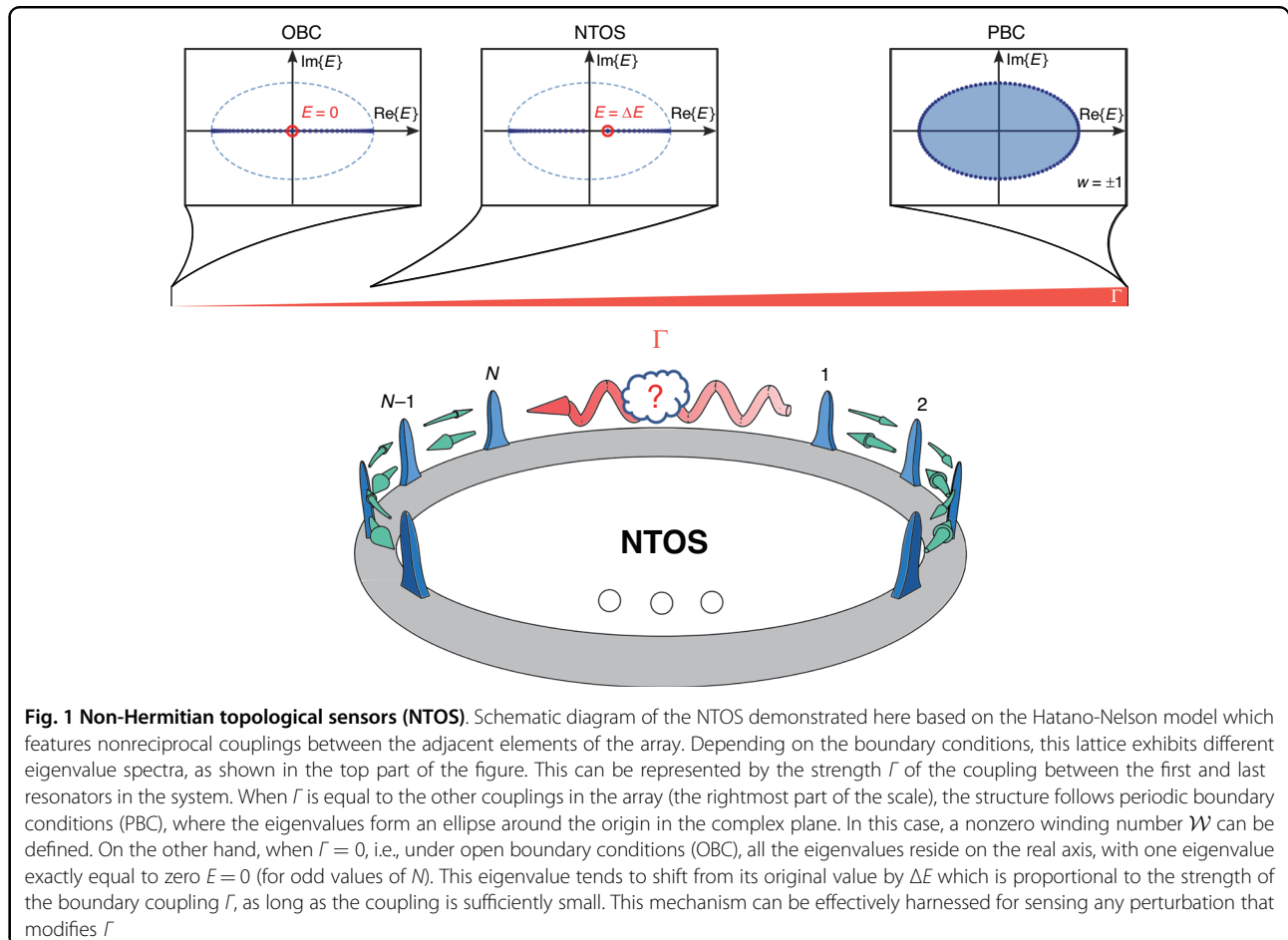
proposed^{26–28}. Dubbed as non-Hermitian topological sensors (NTOS), such devices can exhibit a sensitivity that grows exponentially with respect to the number of lattice sites. Remarkably, unlike typical non-Hermitian sensing schemes, this boosted sensitivity can exhibit robustness against undesirable fluctuations within the underlying lattice that do not affect the boundary conditions. In addition, the response of NTOS to its input is in the form of a linear amplification, in contrast to other non-Hermitian sensing schemes where the system response is nonlinear. Despite intense activities in the field of non-Hermitian topology, an experimental observation of this enhanced sensitivity in photonic arrangements has so far remained elusive. Here, we experimentally demonstrate this peculiar behavior in a network of photonic time-multiplexed resonators. Using non-Hermitian topological lattices with sizes ranging from $N=7$ to $N=23$, we experimentally demonstrate the characteristic exponential growth of the sensitivity associated with NTOS. In addition, we show that this extraordinary response arises exclusively due to the cooperative interplay between non-Hermiticity and topology, something that is absent in other Hermitian topological settings.

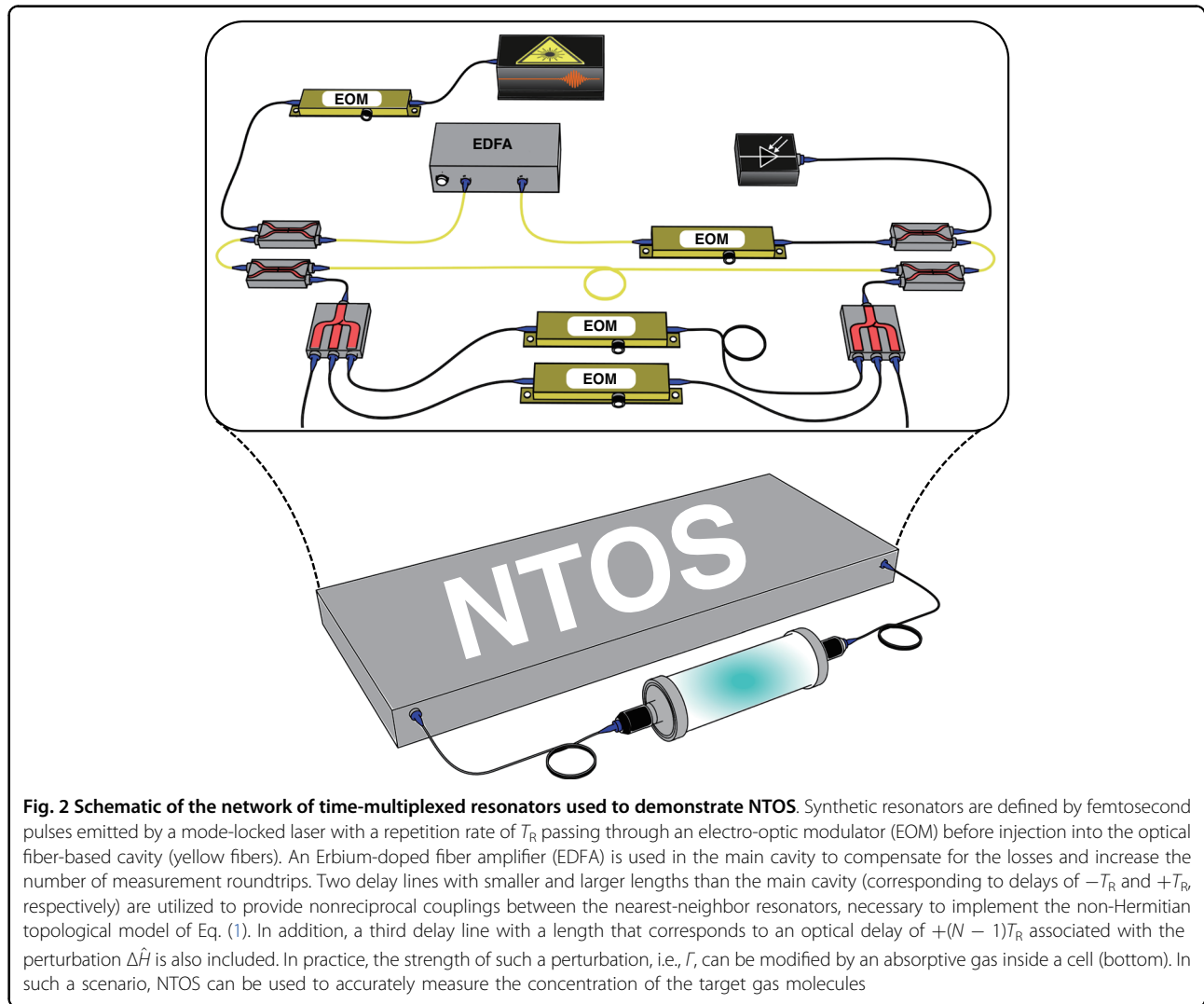
Results

For our realization of NTOS, we consider the Hatano-Nelson (HN)²⁹ model as described by the Hamiltonian:

$$\hat{H}_{\text{HN}} = \sum_n t_R \hat{a}_{n+1}^\dagger \hat{a}_n + t_L \hat{a}_n^\dagger \hat{a}_{n+1} \quad (1)$$

where $\hat{a}_n^{(\dagger)}$ is the annihilation (creation) operator associated with site n while t_R, t_L represent the nonreciprocal right and left nearest-neighbor couplings within the lattice. For a finite lattice, the Hamiltonian of Eq. (1) can exhibit a multitude of spectral behaviors, depending on the associated boundary conditions (Fig. 1). In particular, when the lattice is arranged in a uniform fashion with periodic boundary conditions (PBC), the set of eigenvalues form a closed loop in the complex plane with a nonzero winding around the origin (Fig. 1), and the eigenstates are uniformly distributed across the lattice. We would like to emphasize that here, since the coupling mechanism between lattice elements are dissipative³⁰, the real part of the system eigenvalues represent dissipation while the imaginary part corresponds to phase/frequency shift (see Supplementary part 1). On the other hand, when the structure is terminated with open boundary



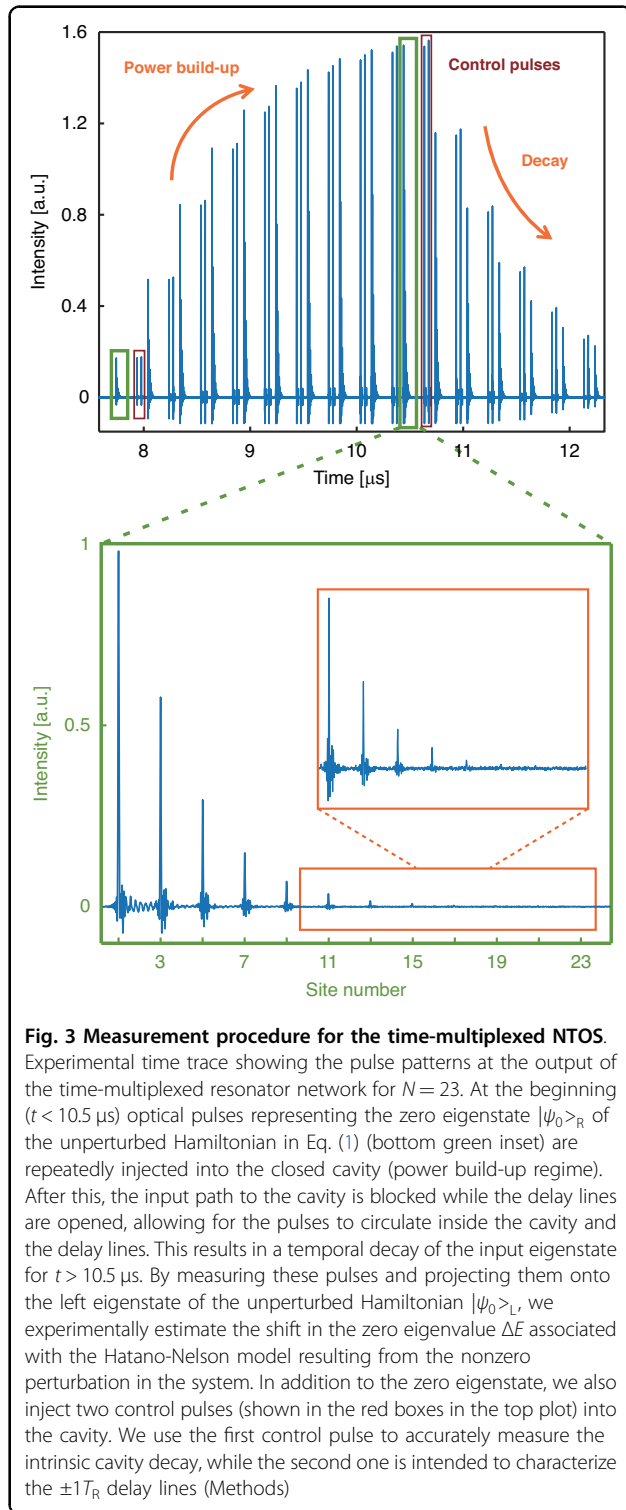


conditions (OBC), the resulting spectrum is entirely real (Fig. 1). This corresponds to the case where all the eigenstates become localized near one edge of the system, known as the non-Hermitian skin effect³¹. Furthermore, under such OBC, provided that the number of elements in the lattice is odd $N = 2k + 1$, the Hamiltonian $\hat{H}_{\text{HN}}$ always possesses an eigenstate $|\psi_0\rangle_R$ with an eigenvalue equal to zero (i.e., a zero-mode).

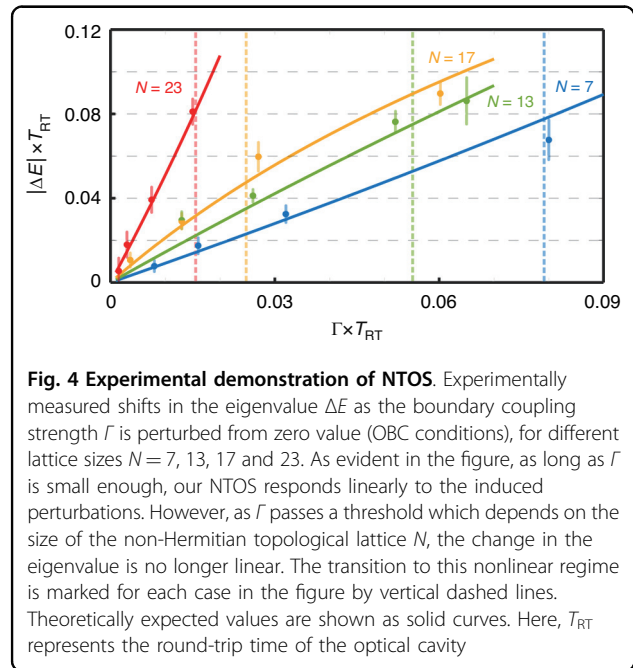
To experimentally demonstrate NTOS, we use a time-multiplexed photonic resonator network depicted schematically in Fig. 2. The network consists of a main fiber loop which supports N resonant pulses separated by a repetition period, T_R . Here, each individual pulse represents a single resonator associated with the annihilation (creation) operators $\hat{a}_j^{(i)}$ in Eq. (1). To realize the non-reciprocal couplings t_R and t_L , we use two delay lines to dissipatively couple nearest-neighbor pulses. Each delay line is equipped with intensity modulators that control the strengths of such couplings (see Fig. 2). To induce the

perturbation signal, we consider a change in the lattice of the form $\Delta\hat{H} = \Gamma\hat{a}_N^\dagger\hat{a}_1$ which shows a small deviation from the OBC configuration. In response to this perturbation, the unperturbed eigenstate $|\psi_0\rangle_R$ will now change to $|\psi(\Gamma)\rangle_R$ associated with a new eigenvalue that shifts from the zero point by ΔE along the real axis (see Fig. 1). To experimentally implement the perturbation $\Delta\hat{H}$ we use a third delay line which couples the first pulse to the last one in a non-reciprocal fashion. The strength of this coupling is then modulated accordingly to provide different values of the perturbation strength Γ . In a practical scenario, such a perturbation can represent the concentration of an absorptive gas inside a cell that is placed in the path of this third delay line (Fig. 2). Our NTOS can then be used to accurately measure small changes in the concentration of the target gas molecules.

In our experiments, we first initialize the system by shaping the amplitudes and phases of the input pulses to represent the zero eigenstate $|\psi_0\rangle_R$ associated with the



Hamiltonian in Eq. (1). Figure 3 shows an example of such pulses in the experiments concerning $N=23$ time-multiplexed resonators (the green inset depicts the zero eigenstate $|\psi_0\rangle_R$). In order to increase the power of the pulses in the measurements, we repeatedly inject this



pulse pattern into the closed cavity (with closed delay lines) which results in building up power inside the cavity (Fig. 3 for times $< 10.5 \mu\text{s}$). After this initialization, the input to the cavity is blocked so that the pulses start to circulate through the cavity and the delay lines according to the discrete-time evolution corresponding to the Hatano-Nelson model. Subsequently, at each time step that is defined by the integer multiples of the cavity round-trip time, we project the state of the network on the left eigenstate of the unperturbed Hamiltonian $|\psi_0\rangle_L$. The perturbed eigenvalue can now be estimated from the decay rate of this projection per cavity round-trip (see Methods and Supplementary parts 2–4). In addition to the zero-mode, we also inject two control pulses into the unused time slots of the main cavity (red boxes in Fig. 3). The first control pulse is left uncoupled to other time slots of the network and is utilized to accurately measure the intrinsic cavity decay rate (corresponding to the zero-mode decay rate of the HN model). Using this as a reference, we then measure ΔE by calculating the difference between the perturbed eigenvalue and the unperturbed one. We use the second control pulse to accurately characterize the throughput of the $\pm 1T_R$ delay lines which set the nearest-neighbor nonreciprocal right and left couplings t_R and t_L within our HN lattice (see Methods).

Figure 4 displays experimentally measured changes in the eigenvalues obtained for HN lattices with different sizes and for different perturbation strengths together with simulated results. For perturbations well below a critical value $\Gamma \ll \Gamma_C$, the NTOS exhibits a linear response with respect to the input parameter. However, for larger

perturbations, the shifted eigenvalue associated with $|\psi(\Gamma)\rangle_R$ is no longer real, signaling a crossover to the PBC where the sensor response is no longer linear¹⁹. By increasing the perturbation further, the non-Hermitian skin effect breaks down and the eigenstates are no longer exponentially localized at the edge of the structure. Since the performance of the NTOS as a sensor is contingent upon this localization, it is crucial to avoid this non-Hermitian phase transition. Although in the thermodynamic limit Γ_C tends to vanish, our analytical results show that for finite lattices its value remains nonzero and scales exponentially with N . In the Supplementary part 5 we evaluate Γ_C for different values of the lattice size N implemented in our experiments. In order to fully characterize our NTOS, we applied perturbations in a wide range of strengths spanning both below and above the aforementioned critical coupling. As shown in Fig. 4, the experimentally measured results exhibit a linear system response to small Γ . For larger inputs, the sensor response eventually becomes nonlinear, hence setting the dynamic range of our demonstrated NTOS. Hence, there is a fundamental trade-off between the enhanced response and the dynamic ranges attainable by NTOS as the size of the non-Hermitian lattice grows. For our experiments, the requirement to reliably generate such small perturbations limited our demonstration of NTOS to lattices with the number of sites smaller than $N \approx 23$.

Discussion

To evaluate the performance of NTOS, we calculated the sensitivity defined as $S \equiv \partial E / \partial \Gamma$ using our measurement data in the small parameter regime $\Gamma \ll \Gamma_C$. Figure 5 shows theoretically expected values along with experimental results for different lattice sizes N . As evident in this figure, the sensitivity of the NTOS grows exponentially with the size of this non-Hermitian topological system. These results are consistent with theoretical predictions based on perturbation theory. As mentioned earlier, a unique aspect of NTOS is its boosted response to perturbations in the boundary conditions while suppressing undesirable fluctuations in other parameters defining the lattice. In particular, it can be shown that random perturbations in the non-reciprocal couplings t_L and t_R tend to have negligible effects on the output response (please see Supplementary Section 1). Another possible disorder in the lattice can arise due to random changes in the losses associated with different lattice sites. As shown in the Supplementary Section 1, although their effect is not completely nullified, the NTOS does not amplify such perturbations and tends to suppress them as the number of lattice sites N becomes larger. It should be emphasized that our implementation based on time-multiplexing of resonators further minimizes these undesirable loss fluctuations. Remarkably, the exponential

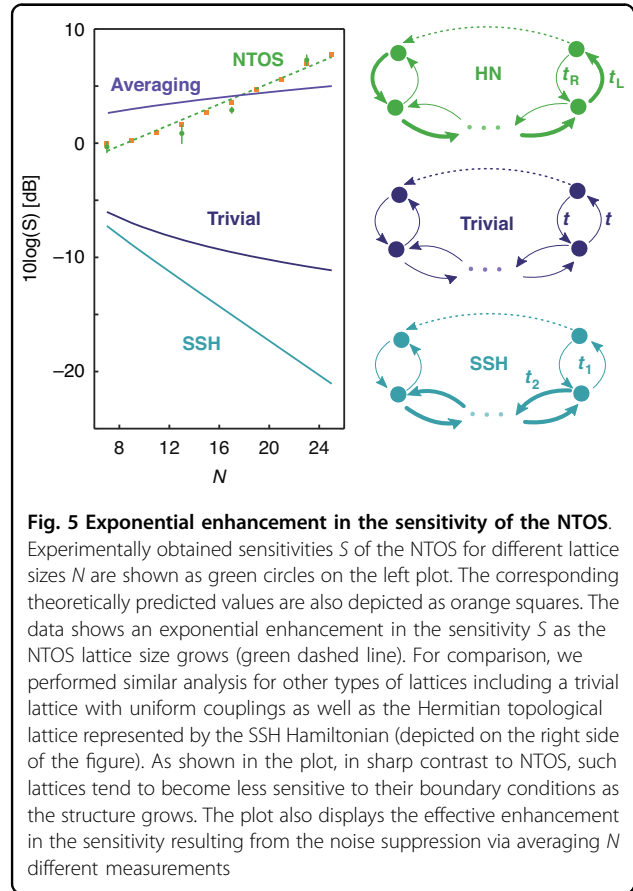


Fig. 5 Exponential enhancement in the sensitivity of the NTOS.

Experimentally obtained sensitivities S of the NTOS for different lattice sizes N are shown as green circles on the left plot. The corresponding theoretically predicted values are also depicted as orange squares. The data shows an exponential enhancement in the sensitivity S as the NTOS lattice size grows (green dashed line). For comparison, we performed similar analysis for other types of lattices including a trivial lattice with uniform couplings as well as the Hermitian topological lattice represented by the SSH Hamiltonian (depicted on the right side of the figure). As shown in the plot, in sharp contrast to NTOS, such lattices tend to become less sensitive to their boundary conditions as the structure grows. The plot also displays the effective enhancement in the sensitivity resulting from the noise suppression via averaging N different measurements

enhancement of the sensitivity is known²⁷ to arise in scenarios where the bulk non-Hermitian lattice possesses a nonzero topological winding number $\mathcal{W}$ defined as

$$\mathcal{W} = \frac{1}{2\pi i} \int_{-\pi}^{\pi} dk \frac{\partial}{\partial k} \log \{ \det [H(k)] \} \quad (2)$$

Here, $H(k)$ denotes the Bloch Hamiltonian associated with the implemented lattice under PBC conditions. To corroborate this, we simulated the behavior of other types of lattices when subjected to the same perturbation Γ in their boundary conditions as the NTOS studied here. We first consider the limiting case of the Hamiltonian in Eq. (1) where the nearest-neighbor couplings become reciprocal $t_R = t_L$, resulting in a trivial system $\mathcal{W} = 0$. As shown in Fig. 5, the sensitivity of a sensor implemented using a uniform lattice tends to deteriorate as $1/N$ with respect to the number of array elements (see Supplementary part 6). As a second example, we choose a Hermitian, but topologically non-trivial lattice, namely that associated with the Su-Schrieffer-Heeger (SSH) model³². When properly terminated, such a lattice also supports a pair of topological edge states that are localized in the open ends of the structure, in a way similar to the

NTOS constructed in our experiments. However, unlike NTOS, the SSH Hamiltonian exhibits a trivial non-Hermitian winding number according to Eq. (2). For this system, it can be shown (see Supplementary part 6) that the sensitivity of the eigenvalues associated with such Hermitian edge states are in fact exponentially *insensitive* to the changes in the boundaries of the array as N grows (Fig. 5). These results hence confirm that the unusual enhancement in the sensing response observed in our experiments arises uniquely due to the synergy between non-Hermiticity and topology.

In summary, we have experimentally demonstrated enhanced sensitivity by non-Hermitian topological amplification. For HN lattices comprised of different numbers of elements, we characterized the response of the system as the shift in one of its eigenvalues as the boundary conditions change. While this response tends to saturate for perturbations larger than a critical limit, it tends to be linear for smaller ranges. The sensitivity parameter calculated using experimental data clearly exhibits an exponential growth with the lattice size N , in agreement with theoretical predictions. Using examples of other types of lattices, we showed that this peculiar enhancement arises due to the collaborative effect of non-Hermiticity and topology, something that does not occur for instance in Hermitian topological systems. As indicated in various recent studies^{26–28,33,34}, another distinguishing feature of NTOS is to boost the sensitivity without suffering from undesirable noise enhancements which could notoriously limit the application of topologically trivial non-Hermitian sensing schemes (please see Supplementary Section 7). Consequently, our work sheds light on novel avenues where the synergistic cooperation between non-Hermiticity and non-trivial topology can unlock functionalities that are otherwise unattainable in physical systems lacking these combined features. We would like to emphasize that the underlying principle of the enhancement in our study is topological amplification in a classical context, due to the specific implementation in our experimental setup. Demonstrating similar effects in the quantum regime (for instance using the model proposed in ref.²⁶) is an exciting prospect that could be the subject of future studies.

Materials and methods

Experimental procedure

To realize non-Hermitian topological sensors (NTOS), we construct the fiber-based time-multiplexed resonator network shown in Fig. 2. This network consists of a main cavity (yellow fiber) and three optical delay lines (blue fiber). We populate this network with optical pulses separated by a repetition period $T_R \approx 4$ ns which are generated by a mode-locked laser optical frequency comb source (MenloSystems FC1500-250-WG), and we choose the lengths of the delay lines to introduce couplings between these pulses. These

delay lines are assembled from optical rails and rail carriers (Thorlabs RLA2400, RC4) and FC/APC fiber collimation packages. The $\pm 1T_R$ delay lines produce couplings between nearest-neighbor pulses in the cavity, while the $+(N-1)T_R$ delay line, which is where we introduce perturbations, couples the first pulse in our synthetic lattice to the final pulse. While the main cavity can support up to 74 pulses, we use the $+(N-1)T_R$ delay line to set the size of the lattice under study, and we do not excite the unused time slots in the main cavity.

Prior to an experiment, we calibrate the electro-optic modulators (EOMs, IXblue MXAN-LN-10) in the network using the calibration procedure described in Supplementary part 3. We calibrate the EOM between the laser and the main cavity to carve the zero-mode of the unperturbed Hatano-Nelson lattice from the pulse train of the laser, while we calibrate the modulators in the $\pm 1T_R$ delay lines to implement the Hatano-Nelson model's asymmetric couplings. The $+(N-1)T_R$ delay line also contains two EOMs (not shown in Fig. 2), which control the strength of the perturbation between the first and final sites of the HN lattice. We calibrate the throughput of these modulators to set the perturbation strength for any given experiment.

After completing our calibration, we begin an experiment by injecting the Hatano-Nelson zero-mode into the network for 10 roundtrips, which allows the power in the zero-mode to resonantly build up within the cavity. We implement a Pound-Drever-Hall (PDH) locking scheme to lock the cavity and the delay lines to the comb source by using output taps that are fed into an adjustable gain fiber-optic receiver (New Focus Model 2053), providing an electronic locking signal input to a proportional-integral derivative (PID) controller (Red Pitaya STEMLab 125-14). During the injection phase, we leave the IMs in the $\pm 1T_R$ delay lines biased to minimum throughput so that we do not couple neighboring pulses through these delay lines. After 10 roundtrips, we stop injecting the zero-mode and we turn on the couplings in the $\pm 1T_R$ delay lines. We save a trace of the cavity ring-down by detecting individual pulses using a biased detector (Thorlabs DET08CFC) read by an oscilloscope (Tektronix MSO6B), and we repeat this measurement on the order of 100 times to generate statistics for our data analysis. The control signals for the EOMs and the oscilloscope is generated by an FPGA (Xilinx Zynq UltraScale+ RFSoC).

In addition to injecting the zero-mode into our network, we also inject a single pulse (first control pulse in Fig. 3) into one of the unused time slots of the main cavity. We leave this single pulse uncoupled to the surrounding time slots so that this pulse decays at the intrinsic decay rate of just the main cavity. In the absence of the perturbation, this is the same decay rate that we would expect for the zero-mode of the Hatano-Nelson model. Therefore, this auxiliary pulse acts as a reference from which we can



extract the change in the decay rate of the zero-mode due to the perturbation.

Finally, we inject a second single pulse (second control pulse in Fig. 3) into our time-multiplexed network that we use for accurately measuring the couplings associated with the $\pm 1T_R$ delay lines. For this pulse, we open the delay lines so that the neighboring time slots next to this pulse are populated with light pulses with amplitudes proportional to the nearest-neighbor nonreciprocal right and left couplings t_R and t_L within the HN lattice.

Data analysis

As illustrated in Fig. 3, the state of the time-multiplexed network in every roundtrip is represented by the amplitudes of the pulses within the cavity time slots which define different site numbers in the Hatano-Nelson lattice. In our experiments, we record these time traces for 100 instances and use the average of these traces to measure the state of the network $|\psi(mT_{RT})\rangle$ in successive roundtrips $m = 1, 2, \dots$, where T_{RT} represents the roundtrip time of the optical cavity. We then project this state into the left eigenstate of the unperturbed HN model to define $P(m) = \langle \psi_{0L} | \psi(mT_{RT}) \rangle$. From here, we estimated the decay rate of $P(m)$ per cavity roundtrip to measure the response of NTOS defined as the quantity $|\Delta E \times T_{RT}|$ that is reported in Fig. 4.

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Author contributions

All authors contributed to the writing of this manuscript.

Data availability

The data used to generate the plots and results in this paper is available from the corresponding author upon reasonable request.

Conflict of interest

A.M. has financial interest in PINC Technologies Inc., which is developing photonic integrated nonlinear circuits. The remaining authors declare no competing interests.

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